1. *Résolution de l' équation* $Z^7 = 1$

$$\forall p \in C \qquad Z^7 = 1 \Leftrightarrow \exists k \in \{ 0 , 1 ,\ldots\ldots 6 \} : z = \omega_k = \exp(\frac{2ik\pi}{7})$$

2. *Calcul de la somme* : $\sigma_p = \sum_{k=0}^{6} \omega_p^k$

$$\forall p \in \{ 0 , 1 ,\ldots 6\} : \sum_{k=0}^{6} \omega_p^k = \frac{1-\omega^7}{1-\omega} = 0$$

3. *Calcul de la somme* $\sigma'_p = \frac{\omega_p}{1+\omega_p^2} + \frac{\omega_p^2}{1+\omega_p^4} + \frac{\omega_p^3}{1+\omega_p^6}$

$$\sigma'_p = \frac{\omega(1+\omega^4)(1+\omega^6)+\omega^2(1+\omega^2)(1+\omega^6)+\omega^3(1+\omega^2)(1+\omega^4)}{(1+\omega^2)(1+\omega^4)(1+\omega^6)} = -2$$

$$\forall p \in \{ 0 , 1 ,\ldots 6\} \qquad \frac{\omega_p}{1+\omega_p^2} + \frac{\omega_p^2}{1+\omega_p^4} + \frac{\omega_p^3}{1+\omega_p^6} = -2$$

3. *Calcul de la somme* $\sigma'' = \frac{1}{\cos\frac{2\pi}{7}} + \frac{1}{\cos\frac{4\pi}{7}} + \frac{1}{\cos\frac{6\pi}{7}}$

$$\sigma'' = \frac{2}{\omega_1 + \overline{\omega_1}} + \frac{2}{\omega_1^2 + \overline{\omega_1^2}} + \frac{2}{\omega_1^3 + \overline{\omega_1^3}} = 2\left(\frac{\omega_1}{\omega_1^2+1} + \frac{\omega_1^2}{\omega_1^4+1} + \frac{\omega_1^3}{\omega_1^3+1}\right) = -4$$

et

$$\frac{1}{\cos\frac{2\pi}{7}} + \frac{1}{\cos\frac{4\pi}{7}} + \frac{1}{\cos\frac{6\pi}{7}} = -4$$